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Original Research Article

Security Prices as Markov Processes: Some Evidence on the Random Character of Stock Prices in the Nigerian Bourse

E. J. Idolor[1], O. Okoro[2] & G. B. Abdul[3]
[1] Department of Banking and Finance, Faculty of Management Sciences, University of Benin, Benin City, Nigeria.
[2,3] Department of Banking and Finance, Faculty of Business Administration, University of Nigiera, Nsukka, Enugu Campus, Enugu State, Nigeria.

*For correspondence, email: eseoghene.idolor@uniben.edu

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Abstract

This paper provides a method of Markovian analysis of security prices. In the study we examine four securities selected from the finance and banking segment of the Nigerian bourse for a time frame ranging from January 4th 2005, to December 4th 2008. In our analysis we define three possible states of nature (rise, drop, stable) with regards to security price change process within a modeled Markovian Framework. Our definition of the possible set of states allows both the magnitude and the direction of change to be incorporated into the analysis. The findings reveal that the Markov Chains did not provide a reliable prediction of security price movements for the period of our analysis. It is therefore recommended that we can only adopt the position that at best Markov Chains (for now) only helps to enrich our understanding of stock price behaviour, as far as the random walk hypothesis is concerned, even if the ultimate goal of prediction still proves difficult and elusive.

Keywords: Markov Chains, Markov Processes, Random Walk, Stock Price Transition

1.0 INTRODUCTION

The prediction of stock price behaviour has been a major challenge to stakeholders in the literature. Many attempts have been made to predict stock price behaviour in the past. Analysts have used fundamental and technical approaches and more tools are being evolved in the literature to deal with

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this aspect of the stock market. All the attempts are to see if an investor can beat the market and reap a windfall. The success of such analytical tools could lead to an upward trend in the stock market and further lead to market vibrancy and economic growth and development. Some of these analytical tools have had some successes in terms of long-term prediction of stock price behaviour. For example, the Markov Chain approach has attracted latter day analysts and has been adjudged a possible tool of the future in both developed and developing economies. The use of Markov Chains has received a new impetus, due to an increasing attempt by researchers to develop new tools for predicting stock price behavior, and is at the front burners of stock price analysis in the literature.

Markov theory could be of importance in the analysis of security prices in two ways: firstly, it can be utilised as an important means for making probabilistic assumptions about the direction of future equity price movements or returns levels; and secondly it can provide a more robust mathematical extension and analysis of the random walk hypothesis. It therefore could serve as an alternative to the much popular regression forecasting models utilised in much of the extant literature, and other techniques utilized by technical analysts, in the analysis of stock price behaviour. Ryan (1973) posits that Markov theory deals with the movement of a probabilistic system from its prior state to another. In instances concerning a mathematical sequence of observations on security returns or prices, the states of the Markovian system could readily be regarded as the set of all sequence of observations on stock prices and returns. Theoretically, the observed states being studied can be regarded as one consisting of the set of all possible equity returns or prices for a particular security. The possible states so defined in the system is usually regarded to be infinite, thereby making it convenient in the extant literature to group security returns or prices into ranges, or particular classes for easy analysis. Whenever equity prices or returns are referred to as portraying a Markovian process, it simply suggests that certain theorems relating to the theory of Markovian processes can possibly be utilised to enable us resolve pertinent questions bordering on the future the possible future price levels of given securities (Ryan, 1973).

For the current analysis, the Markovian model we studied is a first – order chain. In particular, the chain consists of a finite number of states and a finite number of points for which our observations were made. For the current study, it is shown that under a fairly general and partial-adjustment Markov Chain model of stock price determination, any price movements that do not particularly display the random walk characteristic may be readily interpreted as purely conforming to a Markov process. This is in line with similar studies like Anderson and Goodman (1957), Chakravarti, Laha and Roy (1967); Bhargava, (1962), Fielitz and Bhargava (1973), Eriki and Idolor (2009) and Idolor and Braimah (2015).

Against this background, the purpose of this study is to attempt an exploratory analysis of some possible means in which Markov processes can be readily utilised in security price and returns movements analysis in the capital market. The study posits that successive price or returns movements of securities may readily be hypothesized as portraying Markovian tendencies that could provide valuable insight or information to local and international financial asset managers. Dryden (1969) and Ryan (1973) conducted investigations, in which aggregate and individual stock data on United Kingdom share prices was analysed within a Markovian framework, and which indicated that it might be fruitful to apply the Markov model to more disaggregated data, specifically to individual stock price data.

2. LITERATURE REVIEW AND HYPOTHESIS DEVELOPMENT

In analysing Markov models; the occurrence of future states of existence of the system in the Markovian model, is often mathematically depicted as being dependent on the immediately preceding state of the system, and, only on it (Taha, 2001). If $t_0<t_1 <\dots t_n$ (n=0, 1, 2,…), it mathematically represents successive points in time. Therefore, the family of random variables $\{\xi_{tn}\}$ is regarded as a Markov process if it clearly possesses the following Markovian property:

$$\mathbf{P}\ \{\xi_{tn} = X_n|\{\xi_{tn-1} = X_{n-1},\dots, \{\xi_{t0} = X_0\} =$$

$$\mathbf{P}\ \{\xi_{tn} = X_n|\{\xi_{tn-1} = X_{n-1}\}$$

This holds for all possible values of ξ_{t0}, $\xi_{t1},\dots, \xi_{tn}$. **(1)**

The probability $P_{xn-1}, X_n = P\ \{\xi_{tn} = X_n|\xi_{tn-1} = X_{n-1}\}$ can be referred to as the transition probability of the system; it represents all the conditional probabilities of the Markov system being or remaining in X_n at t_n; provided it was initially in X_{n-1} at t_{n-1} (X indicates the states of the system while t is the period or time of its occurrence). This mathematical representation can typically also be referred to as one-step transition basically because it describes the Markov transition from t_{n-1} to t_n; while m-step transition probabilities of the system can be mathematically portrayed as shown in equation 2.

$$P_{xn}, X_{n+m} = P\ \{\xi_{tn+m} = X_{n+m}\ |\xi_{tn} = X_n\} \quad \mathbf{(2)}$$

Markov Chains

Markov chains typically referred to as Markovian models are a unique class of mathematical techniques applicable to quantitatively inclined decision problems. Markov chain derives its Name from a Russian Mathematician who is credited with the development of the method. Markov chains could serve as veritable tools in examining frequency with which customers continue to patronise certain brands of product or alternately the frequency of switching to others. In the extent literature, it is generally assumed that people do not arbitrarily shift the brands they purchase randomly, but rather purchase brands in the future which reflects preferences, in the not too distant past. Other areas of applications of Markovian models are in relation to manpower planning models, security behaviour modeling, and, bad debts determination models and credit management models (Agbadudu, 1996). Markovian models are basically regarded in this light as a sequence of states of a probabilistic system that displays Markovian properties. For every given time of analysis, the probabilistic system could change from the prior state it was in before or remain unchanged. Changes in the states of nature are referred to as transitions. For a series of states to display the Markovian property, it indicates that future states of the probabilistic system are conditionally independent of prior states given their current states of nature (Obodos, 2005).

Markovian chains are therefore regarded as a series of events in which the probabilities of occurrences for every event depends upon their immediately preceding events. Theoretically, they could also be referred to as a first-order Markovian Chain Process, or first-order-Markovian. For finite Markov Chains, it is assumed that the sequence of events have some of the following Markovian properties: (i) outcomes for successive experiment are only one of a finite number of possible outcomes $a_1, a_2,..., a_n$, (ii) probability of occurrence of outcome a_j for every given experiment may not necessarily be independent of the outcomes of previous successive experiments, but rather, it is very much largely dependent at the very most upon the outcome, a_i of its immediately preceding experiment, (iii) there exist given numbers P_{ij} that usually are representative the probability of outcome a_j for each experiment, as far as outcomes a_i have been seen to have occurred in preceding experiments. These are therefore the probabilities of moving from initial position i to the new position j from a single step or movement. This movement is mathematically presented as P_{ij}. The outcomes $a_1, a_2, ..., a_n$ are called states and the numbers P_{ij} are called transition probabilities. The number of experiments, or number of movements are often referred to as steps. In some instances, the probability distribution of the initial state may be given, but this may not be very necessary when determination of steady state equilibrium is the focus (Agbadudu, 1996). The number P_{ij} which represents the probability of moving from state a_i to state a_j in one step can readily be portrayed in a matrix form called a transition matrix. This matrix for general finite Markov Chain processes exhibiting states $a_1, a_2, ..., a_n$ are mathematically denoted as follows:

$$P = P_{ij} = \begin{matrix} P_{11} & P_{12} & \dots & P_{1n} \\ P_{21} & P_{22} & \dots & P_{2n} \\ P_{n1} & P_{n2} & \dots & P_{2n} \end{matrix} \quad (3)$$

Here, the sum of the elements of each row of the matrix P is 1. This is because the elements in each row represent the probability for all possible transitions (or movements) when the process is in a given state. Therefore, for state a_i, i = 1,2,..., n the transition probabilities is given as follows:

$$\sum_{j=1}^{n} P_{ij} = 1 \qquad (4)$$

If we let $E_1, E_2, ..., E_j$ (j = 0, 1 2, ...) represent the exhaustive and mutually exclusive outcomes (states) of a system at any time. Initially, at time t_o, the system may be in any of these states. Let $a_j^{(0)}$ (j = 0, 1, 2, ...) represent absolute probabilities of the experimental system being in a state E_j at a time t_0. If we further assume that the system displays Markov characteristics, it will yield transition probabilities mathematically presented in equation 5.

$$P_{ij} = P\ \{\xi_{tn} = j | \{\xi_{tn-1} = i\} \qquad (5)$$

This basically is a one-step probability of moving from a prior state i at t_{n-1} to a new state j at $t_{n;}$ for as long as we assume that these probabilities are stationary over time. Transition probabilities from states E_i to states E_j may therefore be readily organised a matrix form as shown in equation 6.

$$P = P_{ij} = \begin{pmatrix} P_{00} & P_{01} & P_{02} & P_{03} \\ P_{10} & P_{11} & P_{12} & P_{13} \\ P_{20} & P_{22} & P_{22} & P_{23} \\ P_{30} & P_{31} & P_{32} & P_{33} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \qquad (6)$$

The matrix P is often referred to as a homogenous transition or stochastic matrix due to the fact that every transition probability P_{ij} are fixed for all the experiments and are as well also independent of time. The probabilities P_{ij} must sufficiently satisfy the conditions mathematically presented in equation 7.

$$\sum P_{ij} = 1 \ for\ all\ ij$$
$$P_{ij} \geq 0 \text{ for all i and j} \qquad (7)$$

Indicating that all row probabilities must add up to one while any single entry in the row or column could have a probability of ≥ 0. The Markov Chain is now defined. The existence of the transition matrix P along with its initial probability $\{a_j^{(0)}\}$ associated with its own prior state E_j completely defines a Markov chain model (Taha, 2001). It is usually common to regard Markovian models at best as models portraying transitional behaviour of probabilistic systems within a range of equal intervals. Theoretically, situations could exist where the length of the interval depends only upon the characteristics of the system and hence may really not be equal. This case is referred to as imbedded Markov Chain.

Classification of States in Markov Chains

In the study of the various aspects of Markov Chains, it is necessary to note that several types of states may exist. In Markovian analysis, it is common to be interested in the behaviour of the system over a short period of time. In this case the absolute probabilities are computed as shown in the preceding section. A more important study involves the long-run behaviour of the system as the number of transitions tends to infinity. In such a case, a systematic procedure that will predict the long-run behaviour of the system is required (Taha, 2001). The following definitions covers some of the popular states in Markov Chains

Irreducible Markov Chain

Markov Chains are defined as irreducible once all possible state E_j can be arrived at from prior state E_j after a certain finite number of transitions for $I \neq j$ with

$P_{ij}^{(n)} > 0$, for $1 \leq n < \infty$ (8)

In this scenario, we say all the states of the Markovian system communicate.

Translent States

If we let T be a subset of S and T^1 its complement. Supposing every state of T^1 can be reached from every prior state of T; and it is readily possible to move from one state of T to a new state of T^1 (but not vice-versa) then state T is called a transient set. A transient state being an element of a transient set (Agbadudu, 1996; Idolor, 2009).

Closed Set and Absorbing States

In Markovian systems, the set C of states is referred to as being closed if the Markovian probabilistic system; once in any of the states of C, persistently remains that state indefinitely. A special example of a closed set is a single state E_j with transition probability $P_{ij} = 1$. In this case E_j is called an absorbing state. All the states of an irreducible chain must form a closed set and no subset can be closed. The closed set C should satisfy all the Markovian conditions and hence could also be studied independently. A state K is referred to as being in an absorbing state if $P_{kk} = 1$. As long as the chain visits K it continues to remain in that state forever. If K satisfies the condition to be regarded as an absorbing state, the first passage probability from i to K can be regarded as the probability of absorption into K, having started at i. Whenever there are more than one absorbing states in a chain, there is the evident possibility of the process being absorbed into one of the states, hence the need to also desirably find the probabilities of absorption. The probabilities are easily obtained mathematically by resolving a system of linear equations. If we assume that a Markov Chain is such that ultimately one of the absorbing states will be reached; if the state is in an absorbing state, the set of absorption probabilities f_{ik} is said to satisfy the following mathematical properties.

$$F_{ik} = \sum_{j=0} P_{ij} F_{jk}, \text{ for all } i = 0, 1,...,M. \quad (9)$$

subject to the conditions that:

$F_{kk} = 1$,

$F_{ik} = 0$ if state i is recurrent and $i \neq k$.

Absorption probabilities are important in "random walks". A random walks are Markovian chains with properties suggesting that if the system is positioned in state i, with a single on step transition, the system could remain at i or very conveniently move to any one of the numerous states that are immediately adjacent to state i (Hilier & Lieberman, 1990).

Ergodic States

These are set of states where once the process moves into it, the process cannot leave it; rather it moves thereafter among states in the set (Agbadudu, 1996, Taha, 2001; Eriki & Idolor, 2009; Idolor & Braimah, 2015). If all the states of a Markov Chain are ergodic, then the chain is irreducible as shown in equation(10).

$$a^{(n)} = a^{(0)}P^n \quad (10)$$

In this case, the absolute probability converges clearly to a particular limiting distribution as as long as $n \to \infty$, whenever this limiting probability distributions are independent of the initial probability distributions $a^{(0)}$ (Taha, 2001).

Classification of Chains

When analysing the various aspects of Markov theory, it is often advisable to note the several forms of Markov chains that are in existence. The chains are very easily classified according to the states of nature they contain (Agbadudu, 1996; Eriki & Idolor, 2009). Two common chains in elementary applications of Markovian theory are Ergodic and absorbing Markovian chains. These chains are briefly discussed in the following subsection section.

Ergodic Chains

Ergodic chains are Markov chains that make it uniquely easy to go between any two states which do not need to be in only one step. This we refer to as a chain consisting of single sets of ergodic states. Two common types of ergodic chains are the cyclic and regular chains. In cyclic chains, each state can only be entered at certain period of intervals. That is, each state is period; with a state being periodic with period t if a return is possible only in t, 2t, 3t,..., steps. This mathematically simply means that $P_{ii}^{(n)} = 0$ whenever n is not divisible by t. Regular chains are-non-cyclic ergodic chains. Regular chains are an important special class of ergodic chains. It is a situation in which some n ($n \geq 2$) P^n has no zero entries. That is, it is possible to go between any two states in n steps.

Absorbing Chains

A chain is often referred to as an absorbing chain, once it has in existence at the very least one absorbing state already and if, from every prior state, it is mathematically feasible in one or more steps to reach an absorbing state (Idolor, 2009)

3.0 METHODOLOGY

In January 2005, the population of deposit money banks in Nigeria was twenty-five (25). We randomly selected, through balloting, four (4) banks quoted in the Nigerian bourse. Our emphasis on the banking sector is justifiable as the Nigerian banking sector has continued to be the most actively traded sector in the bourse. In addition, our time frame was chosen in order to capture the period of the 2005 banking industry reform and consolidation as well the 2008 global financial meltdown which is believed to have triggered a value meltdown in the Nigerian bourse. The equity prices of the four (4) randomly selected banks, collected on a daily basis, ranging from 4th January 2005 to 4th December 2008 constituted our data source for our analysis. We derived our data from Cashcraft Asset Management Limited official website. The only stipulated condition for selecting the securities in our sample is that secondary data on the stock price movements must be available for the entire period covered. This implies that the deposit money banks must have data for the period ranging from 4th January 2005, to, 4th December 2008. The four randomly selected deposit money banks are: Access Bank, United Bank for Africa (UBA), Eco Bank, and First Bank of Nigeria (FBN).

The Market Price Mechanism

Most stock exchanges in the world today run on the Automated Trading System (ATS) (NSE, 2017). Each trading day, Brokers representing the interest of investors go to the floor of the exchange with their bid prices **(Pb)** and offer prices **(Po)** for various quantities **(q)** of stock, **xyz**. One can therefore imagine a situation where brokers come with. $\mathbf{Pb_1}$ for $\mathbf{q_1}$ for investor $_{\mathbf{1}}$ (who is interested in buying into **xyz**), $\mathbf{Pb_2}$ for $\mathbf{q_2}$ for investors $_{\mathbf{2}}$ (who is interested in buying into **xyz**), $\mathbf{Pb_3}$ for $\mathbf{q_3}$ for investor $_{\mathbf{3}}$ (who is interested in buying into **xyz**) and $\mathbf{Po_1}$ for $\mathbf{qd_1}$ for investor $\mathbf{d_1}$ (who is interested in divesting from **xzy**), Po2 for $\mathbf{qd_2}$ for investor $\mathbf{d_2}$ (who is interested in divesting from **xzy**), and $\mathbf{Po_3}$ for $\mathbf{qd_3}$ for investor $\mathbf{d_3}$ (who is interested in divesting from **xzy**)

It will not only be cumbersome but painstaking in trying to settle these interests. Therefore the ATS makes allocation to buyers according to bid prices **(Pb)** with particular reference to offer prices **(Po)**. That is, investor **i** with the highest bid price **(Pbi)** is allocated **qi** quantities of **xyz** security for **Pbi** price, provided **Pbi** is not less than the offer price **Poi**, then the system allocates to the next highest investor **i** with the next highest **pbi** until all the available quantities of the stock **qdi** are allocated at a price not less than their quoted **Poi** (Obodos, 2005; Eriki & Idolor, 2009). Therefore, for any particular trading day, all traded security would have a range of prices for which they changed ownership. These price ranges from high to low and can quite simply be incorporated into the Markov Chains model to either predict the future direction of prices or provide justification for randomness of security prices as we have shown in the study.

Estimation of a Stock's Transition and Initial Probability Matrix

Any Markov process can be completely described by means of its Transition Probability Matrix (TPM). For our study we propose a three state of nature Markov Chain model simply portrayed as rise (r), drop (d) and stable (s); is presented to show three basic possible price movement of a stock on any particular trading day. We also propose that given any previous state it is still possible to migrate to a new state. Thus we can have a rise leading to a rise, a rise leading to stable prices and a rise leading to a drop in prices. This we depicted as rr, rs and rd in Figure 1. Similar conditions are also projected for movements in prices when the prior states are stable or dropping prices. Hence we also have similar scenarios captured in Figure 1 depicted as dr, dd, ds, sr, sd, and ss. For the current model we present also a directed graph shown in Figure 1 to depict the Markovian price movement process. The labelling portrays the probability of moving from state to state.

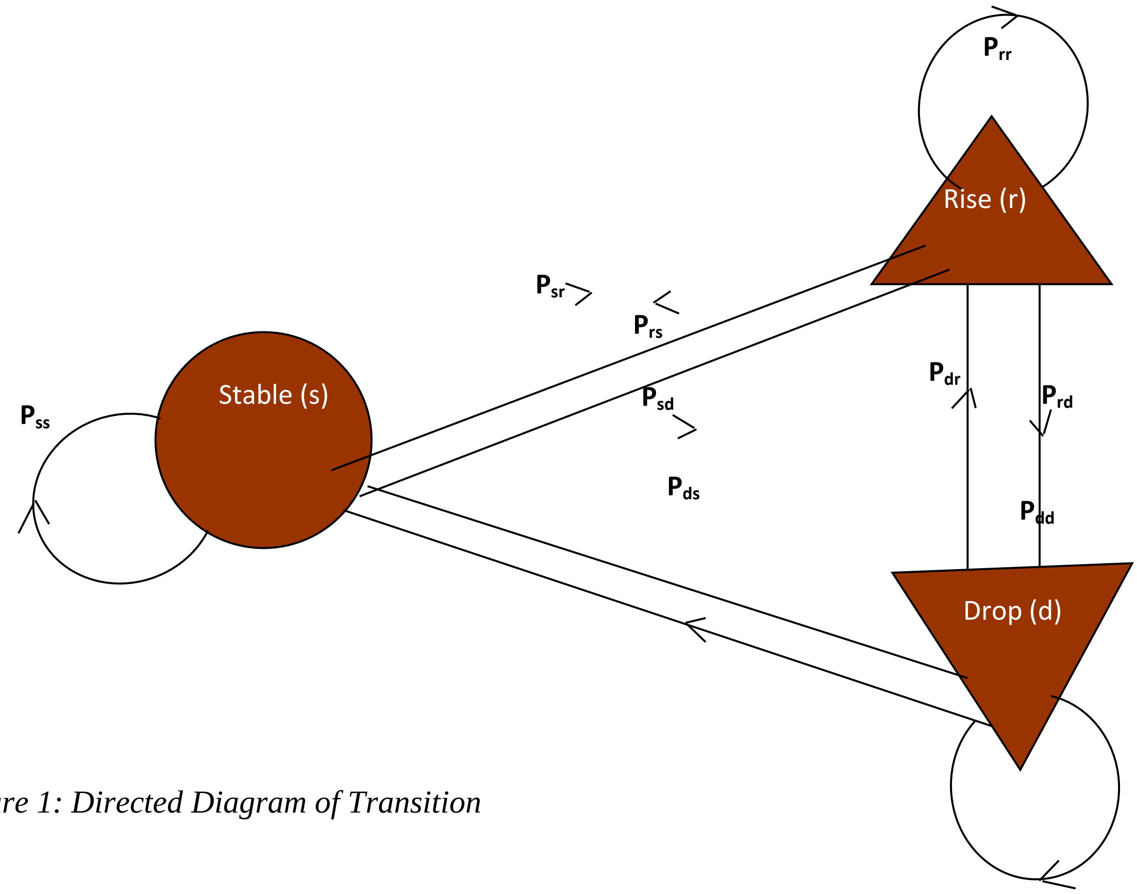

Figure 1: Directed Diagram of Transition

As shown in Figure 1, transition could readily occur from one state to the other, denoting a stable (**s**), rise (**r**), and, drop (**d**) scenario. The probabilities of migrating to the next state is given as $\mathbf{P_i}$ and the sum of probabilities of a necessity must mathematically approximately equal one (**1**). This is depicted as follows:

$$\sum_{i=1} \mathbf{P_i = 1}$$

Therefore, given the initial probability vector $\mathbf{U_o}$, the probabilities of the system migrating to the next state after transition tables are derived are mathematically presented as follows:

$\mathbf{U_1 = U_0.P}$ (Note that $\mathbf{U_o}$ and $\mathbf{P}$ are vectors)

$$\mathbf{U_2 = U_1.P}$$

$$\mathbf{U_3 = U_2.P}$$

$$\mathbf{U_n = U_{n-1}.P}$$

This basically generates the probability of the equity prices transiting from one state to the other. The probabilities of each possible state are mathematically computed using the estimation procedures below:

$$r = \frac{\sum Pr}{\sum Pr + \sum Ps + \sum Pd}$$

$$d = \frac{\sum Pd}{\sum Pr + \sum Ps + \sum Pd}$$

$$\mathbf{s} = \frac{\sum Ps}{\sum Pr + \sum Ps + \sum Pd}$$

$$rr = \frac{\sum \Pr r}{\sum \Pr r + \sum \Pr d + \sum \Pr s}$$

$$rd = \frac{\sum \Pr d}{\sum \Pr r + \sum \Pr d + \sum \Pr s}$$

$$rs = \frac{\sum \Pr s}{\sum \Pr r + \sum \Pr d + \sum \Pr s}$$

$$dr = \frac{\sum Pdr}{\sum Pdr + \sum Pdd + \sum Pds}$$

$$dd = \frac{\sum Pdd}{\sum Pdr + \sum Pdd + \sum Pds}$$

$$ds = \frac{\sum Pds}{\sum Pdr + \sum Pdd + \sum Pds}$$

$$sr = \frac{\sum Psr}{\sum Psr + \sum Psd + \sum Pss}$$

$$sd = \frac{\sum Psd}{\sum Psr + \sum Psd + \sum Pss}$$

$$ss = \frac{\sum Pss}{\sum Psr + \sum Psd + \sum Pss}$$

Using our simple estimation procedures above, a short term daily Markovian model, for three states of nature capturing a rise, drop and stable prices, with transition and initial probability matrix is thus given:

$$\mathbf{U_o} = [\mathbf{U_r} \quad \mathbf{U_d} \quad \mathbf{U_s}] = [\mathbf{P_r} \quad \mathbf{P_d} \quad \mathbf{P_s}]$$

Also,

$$\mathbf{P} = \begin{pmatrix} \mathbf{P_{rr}} & \mathbf{P_{rd}} & \mathbf{P_{rs}} \\ \mathbf{P_{dr}} & \mathbf{P_{dd}} & \mathbf{P_{ds}} \\ \mathbf{P_{sr}} & \mathbf{P_{sd}} & \mathbf{P_{ss}} \end{pmatrix}$$

$\mathbf{U_o}$ = Initial Probability Vector, $\mathbf{P}$ = Transition Probability Matrix, $\mathbf{U_r}$ = $\mathbf{P_r}$ = Probability of rise in security prices, $\mathbf{U_d}$ = $\mathbf{P_d}$ = Probability of drop in security prices , $\mathbf{U_s}$ = $\mathbf{P_s}$ = Probability of security prices remaining stable, $\mathbf{P_{rr}}$ = Probability of security price rise after an initial rise in price, $\mathbf{P_{rd}}$ = Probability of security price dropping after an initial rise in price, $\mathbf{P_{rs}}$ = Probability of security price remaining stable after an initial rise in price, $\mathbf{P_{dr}}$ = Probability of security price rising after an initial drop in price, $\mathbf{P_{dd}}$ = Probability of security price dropping after an initial drop in prices, $\mathbf{P_{ds}}$ = Probability of security price remaining stable after an initial drop in prices, $\mathbf{P_{sr}}$ = Probability of the security price rising after an initial stable price, $\mathbf{P_{sd}}$ = Probability of security price dropping after an initial stable price, $\mathbf{P_{ss}}$ = Probability of the security price being stable after an initial stable price.

Estimation and Testing Procedure

For our estimation and testing, we borrow greatly from the field of mathematics and binary operations. We basically dealt with zeros and ones as popularly used in binary combination and binary mathematics. Here a **1** (one) is used to represent the actual occurrence of an event while **0** (zero) represented nonoccurrence. In sum, this approach is adopted for the over eight hundred daily stock prices, after which a frequency count is taken. Using simple probability and statistical methods quite common to the die and coin tossing problem, a set of formulae is derived for the estimation of the probabilities of the various states (see Anderson & Goodman, 1957; Fielitz & Bhargava, 1972; Obodos, 2005; Idolor, 2009; Eriki & Idolor, 2010; Idolor & Braimah, 2015).

4. ESTIMATION RESULTS AND DISCUSSION OF FINDINGS

We provide empirical results from the Markovian model utilised. Our first objective was to derive the initial probabilities and transition probabilities with the aide of the transition tables developed for the analysis; after which the probabilities of the system moving to the next state was computed for nine consecutive days. These is given in Table 1 to 4. Markov Chains are theoretically assumed to be relevant in the financial analysis of equity prices and returns, since they can be useful for providing informed probable assumptions about security price and returns movement. The superior probability value in the matrices in this regards serve as the basis for our final decision; and may theoretically give a final decision on the general direction of price levels in the bourse for the short run. Once this can be achieved, the probabilities is thus be assumed to provide a fairly conservative projection of equity price and returns in the bourse. This is also true for long run predictions which on the average aims at projecting a long range indication of the prospects of an individual security.

Table 1: Probability of the system moving to the next state (nine days prediction) for Access Bank Plc.

S/N	Date	$U_n = [U_r \quad U_d \quad U_s]$	Price
1	30/06/2008	U_o = [0.2716 0.2633 0.4650]	17.64
2	01/07/2008	U_1 = [0.2782 0.2586 0.4628]	18.52
3	02/07/2008	U_2 = [0.2791 0.2594 0.4609]	18.71
4	03/07/2008	U_3 = [0.2798 0.2600 0.4594]	17.8
5	04/07/2008	U_4 = [0.2803 0.2604 0.4582]	18
6	07/07/2008	U_5 = [0.2807 0.2607 0.4572]	18.3
7	08/07/2008	U_6 = [0.2810 0.2610 0.4564]	18.02
8	09/07/2008	U_7 = [0.2813 0.2612 0.4557]	18
9	10/07/2008	U_8 = [0.2815 0.2613 0.4551]	18.15
10	11/07/2008	U_9 = [0.2816 0.2614 0.4546]	18.01

* Rows may not add up to one exactly because of rounding.

The results from Table 1 shows that the Markovian framework did not give an accurate projection of the actual direction of prices in the bourse for the period under study. Surprisingly, it seems to suggest that prices will continuously remain stable for the nine day period when in actual fact there was a high degree of fluctuations in prices.

Table 2: Probability of the system moving to the next state (nine days prediction) for UBA Plc.

S/N	Date	$U_n = [U_r \quad U_d \quad U_s]$	Price
1	30/06/2008	U_o = [0.1477 0.1125 0.7397]	24.24
2	01/07/2008	U_1 = [0.1483 0.1128 0.7386]	25.4
3	02/07/2008	U_2 = [0.1487 0.1131 0.7376]	25
4	03/07/2008	U_3 = [0.1491 0.1134 0.7367]	24.51
5	04/07/2008	U_4 = [0.1494 0.1136 0.7359]	23.6
6	07/07/2008	U_5 = [0.1497 0.1138 0.7352]	23.99
7	08/07/2008	U_6 = [0.1499 0.1140 0.7345]	23.93
8	09/07/2008	U_7 = [0.1501 0.1141 0.7339]	23.98
9	10/07/2008	U_8 = [0.1503 01142 0.7333]	24.2
10	11/07/2008	U_9 = [0.1504 0.1143 0.7328]	24.1

* Rows may not add up to one exactly because of rounding.

The results from Table 2 shows that the Markovian framework did not provide an accurate prediction of the actual direction of prices for the period under study. The results seems to suggest that prices will continue to remain stable for the nine day period when in actual fact there was a high degree of fluctuations in prices.

Table 3. Probability of the system moving to the next state (nine days prediction) for ECO Bank Plc.

S/N	Date	$U_n = [U_r \quad U_d \quad U_s]$	Price
1	30/06/2008	U_o = [0.2432 0.2314 0.5252]	7.87
2	01/07/2008	U_1 = [0.2430 0.2325 0.5240]	8.26
3	02/07/2008	U_2 = [0.2433 0.2331 0.5229]	8.29
4	03/07/2008	U_3 = [0.2436 0.2335 0.5219]	8.18
5	04/07/2008	U_4 = [0.2439 0.2338 0.5210]	8.17
6	07/07/2008	U_5 = [0.2442 0.2340 0.5202]	8.24

7	08/07/2008	**U_6** = [0.2444 0.2342 0.5195]	8.33
8	09/07/2008	**U_7** = [0.2446 0.2344 0.5189]	7.93
9	10/07/2008	**U_8** = [0.2448 0.2346 0.5183]	7.96
10	11/07/2008	**U_9** = [0.2449 0.2347 0.5178]	7.8

* Rows may not add up to one exactly because of rounding.

The results from Table 3 shows that the Markovian framework did not provide a reliable projection of the actual direction of prices for the period under study. The results seems to suggest that prices will continue to remain stable for the nine day period when in actual fact there was a high degree of fluctuations in prices.

Table 4: Probability of the system moving to the next state (nine days prediction) for First Bank Plc.

S/N	Date	U_n = [U_r U_d U_s]	Price
1	30/06/2008	**U_o** = [0.3960 0.3783 0.2256]	42.77
2	01/07/2008	**U_1** = [0.3950 0.3791 0.2256]	43.8
3	02/07/2008	**U_2** = [0.3947 0.3791 0.2256]	43.45
4	03/07/2008	**U_3** = [0.3946 0.3790 0.2255]	42.44
5	04/07/2008	**U_4** = [0.3945 0.3789 0.2254]	41.99
6	07/07/2008	**U_5** = [0.3944 0.3788 0.2254]	41.91
7	08/07/2008	**U_6** = [0.3843 0.3787 0.2253]	41.45
8	09/07/2008	**U_7** = [0.3942 0.3786 0.2253]	41.96
9	10/07/2008	**U_8** = [0.3941 0.3785 0.2252]	42.01
10	11/07/2008	**U_9** = [0.3940 0.3784 0.2251]	43.9

* Rows may not add up to one exactly because of rounding.

The results from Table 4 shows that the Markovian framework did not give a reliable prediction of the direction of prices for the period under study. Even the few correct predictions were few and far between; and seem to be due mainly to chance. From Table 1-4, it is seen that the Markov Chain model did not give a reliable projection of price movements for the period studied in the short term. This may suggest randomness in stock price behaviour which has been documented in the extant literature often as reflecting the case in the more advanced stock markets. For now the view is held (based on the findings) that Markov Chains cannot be used to predict the direction of security prices in the Nigerian capital market at the moment. Our findings are consistent with some related studies conducted by Fielitz (1969), Fielitz and Bhargava (1973), Mcqueen & Thorley, 1991; Eriki and Idolor (2010) and Idolor and Braimah (2015) where the Markov Chains for daily closing and high price relatives were found to be non-stationary and therefore could not be used for predictive purposes. Our findings generally agree with much of the extant literature that suggests that stock market prices are random and cannot readily be predicted. We therefore posit that Markovian analysis can be used to test the random walk hypothesis, as our findings have shown; and indeed other probabilistic aspects of the different forms of efficient market hypothesis under a uniquely different set of assumptions than is traditionally needed in the extant literature.

5. CONCLUSION AND RECOMMENDATIONS

In the preceding sections, we attempt to provide some veritable means of applying Markovian in theory in equity price analysis. Our findings show that possible states of nature of equity price behaviour can readily be interpreted within the framework of Markovian theory in a way that provides useful information to the portfolio manager. The study examined the stock prices of four (4) randomly selected deposit money banks quoted in the Nigerian bourse. Our main objective was to attempt to project the future price movement or direction of the equities of the selected deposit money banks using their past equity price information. Our findings reveal that equity price movement could not be predicted with the aide of the computed probabilities; and tended to agree with the already established opinion in the empirical literature that stock prices are random. One possible explanation for this occurrence is that different companies are affected at different times by new information that could produce significant differences in the runs and in the large reversal patterns among daily stock prices. For example, some companies might experience price runs as a result of favourable (unfavourable) earning reports, dividend policies, and industry news, while at the same time other companies would not be similarly affected by this information and their daily price change behaviour would then be different. On the other hand, some companies may experience large reversal patterns because of the uncertainty relative to new information, while at the same time other companies would not be similarly affected. Moreover, because new information becomes available at various times, heterogeneous behaviour among stocks is further compounded. While the price behaviour of some groups might be affected by today's news, tomorrow's news could conceivably affect a different group of stocks. In addition standard statistical tests for homogeneity, stationarity and order of the chains in vector process Markov Chains is suggested. If the test shows heterogeneity and nonstationarity in the chains, then it confirms randomness in stock prices and can thus serve as a further piece of evidence in support of the random-walk hypothesis.

The application of Markovian theory in equity price analysis still remains largely an unexplored area in the extant literature, and, probably a very fruitful one as well. Our findings and procedures provided in the study, at the very least, could be taken as a pioneering and rudimentary effort; which in a worst case scenario could be interpreted largely as simple illustrations. Some reasonable research work needs to be conducted on further refinements in the Markovian model such as a Bayesian-type updating of the transition probability matrix (TPM). Additional empirical refinements could also be carried out on the model assumptions before the results from it can be said to yield further statistically strong reliability. We conclude with a consideration of the possible predictive capabilities of a Markov process representation of changes in price when the condition of stationarity and homogeneity in the vector process is satisfied. In stationary, Markov process, tomorrows expected price change given today's price change can be estimated. After several steps, the memory of the starting point is lost. All that remains is the steady-state transition matrix, and, the characteristic vector, that provides the probabilities of being in one new state independent of the prior states.

Thus far, in the development of the mathematical theory of Markov Chains, little is known regarding the empirical analysis of non-stationary models (those with non-stationary transition probabilities). This class of chains is so general that in most cases they are of little predictive value. Even the two-state chain is extremely complicated to analyse, and widely different types of behaviour are possible, depending on the nature of the transition probabilities. Thus, finding some specific manner in which the transition probabilities change is necessary before a detailed study becomes possible. However, the possibility exists that the Markov formulation of the individual process model developed here can be used for predictive purposes if the non-stationarity present in the transition probabilities can be identified and corrected. Efforts along this line, say, by regression analysis, seem to us to be fruitful areas for further research. In this light, we can only adopt the position that at best Markov Chains (for now) only helps to enrich our understanding of stock price behaviour (as far as the random walk hypothesis is concerned) even if the ultimate goal of prediction still proves rather difficult and elusive.

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